education equals

Topic: Arithmetic and Geometric Sequences and Linear Recurrence

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [2, 2, 2, 2, 2: 10 marks]

For the following sequences determine which are arithmetic, geometric or neither.

Provide a reason to support your answer.

a) 1, 1.5, 2.25, 3.375...

b) 5, -5, 5, -5, 5, -5 ...

c) $\frac{7}{8}, \frac{29}{24}, \frac{37}{24}, \frac{15}{8}, \frac{53}{24} \ldots$

d) 300, -60, 12, -2.4, 0.48, -0.096 ...

e) 2, 1, 2, 1, 2, 1 ...

Question Two: [3, 3, 3: 9 marks]

a) A geometric sequence has $T_3 = 4$ and $T_6 = 32$.

i) Determine the recursive rule.

ii) Calculate the 5th term.

b) An arithmetic sequence has $T_3 = -5$ and $T_6 = 4$.

i) Determine the recursive rule.

ii) Calculate the 5th term.

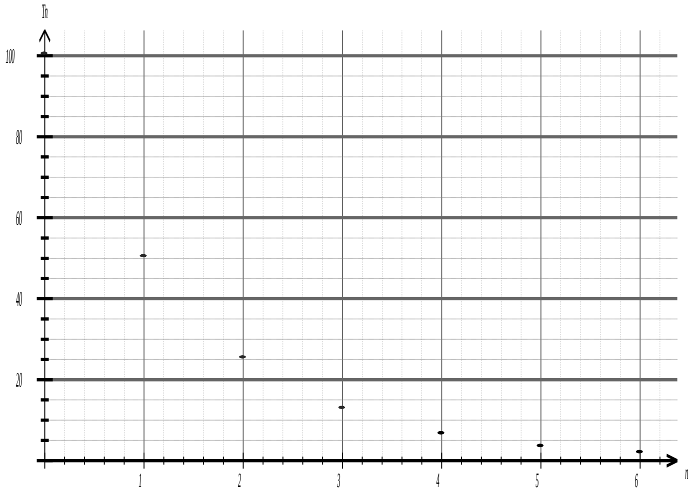
c) For the following sequence determine the recursive rule and term and T_7.

T_1	T_2	T_3	T_4	T_5
4	-8	16	-32	64

Question Three: [3, 2, 2: 7 marks]

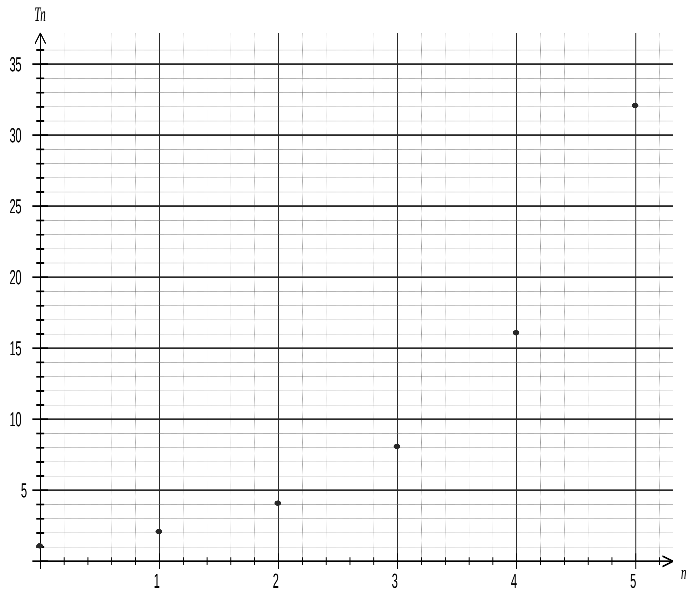
a) An arithmetic sequence has $T_3 = 9$ and a common difference of 2. Determine term 12.

b) The following graph depicts a geometric sequence.



Determine the rule to find the n^{th} term.

c) The following graph depicts a geometric sequence.

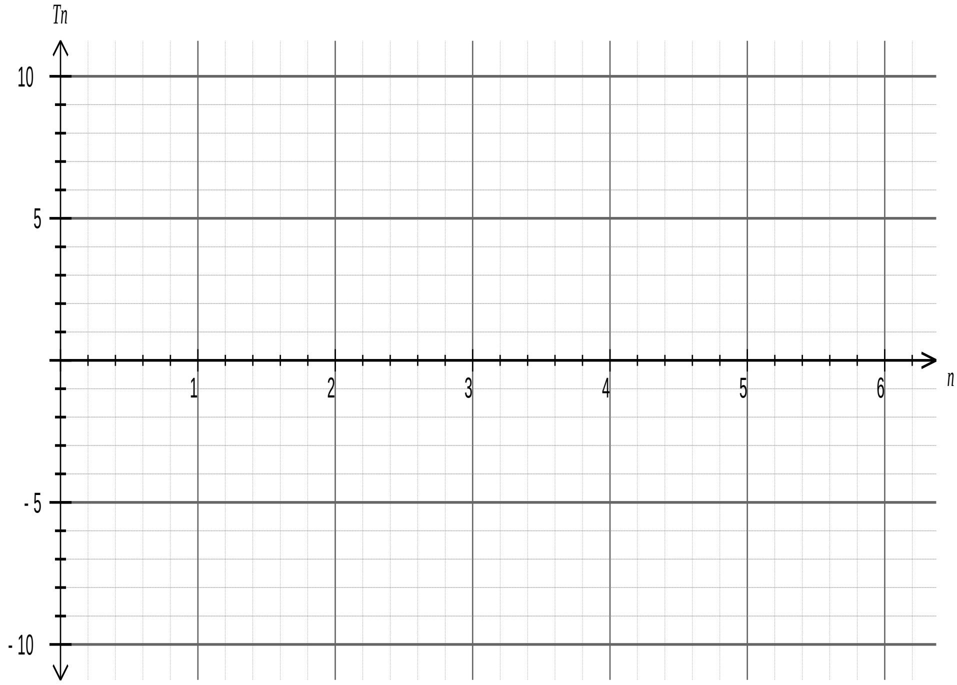


Determine the 8^{th} term.

Question Four: [4, 4: 8 marks]

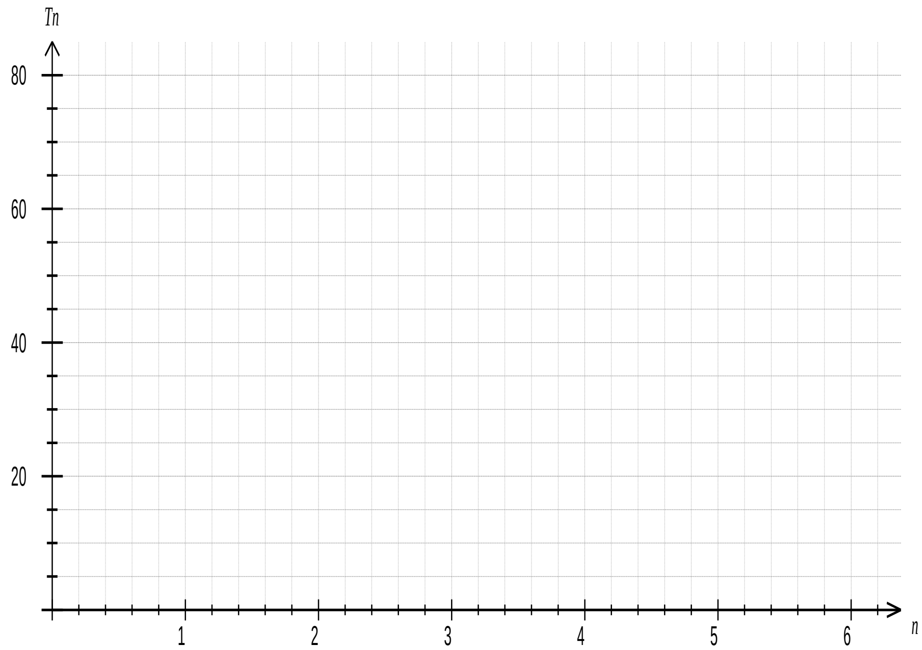
a) Generate the first 5 terms according to the following recursive rule and graph the terms on the axis below.

$T_{n+1} = T_n - 2, \quad T_1 + T_2 = 10$



b) Graph the first 5 terms generated by the following recursive rule on the axis below.

$T_n = 2T_{n-1}\,, \qquad T_1 + T_2 = 15$



Question Five: [2, 2, 2: 6 marks]

Doctors are monitoring the growth of a bacteria in order to help calculate the dosage of medication needed to combat it. The following table shows the growth of the bacteria over several hours.

Population of bacteria (B) in 1000s	Number of hours (t)
100	0
120	1
144	2
172.8	3
207.36	4

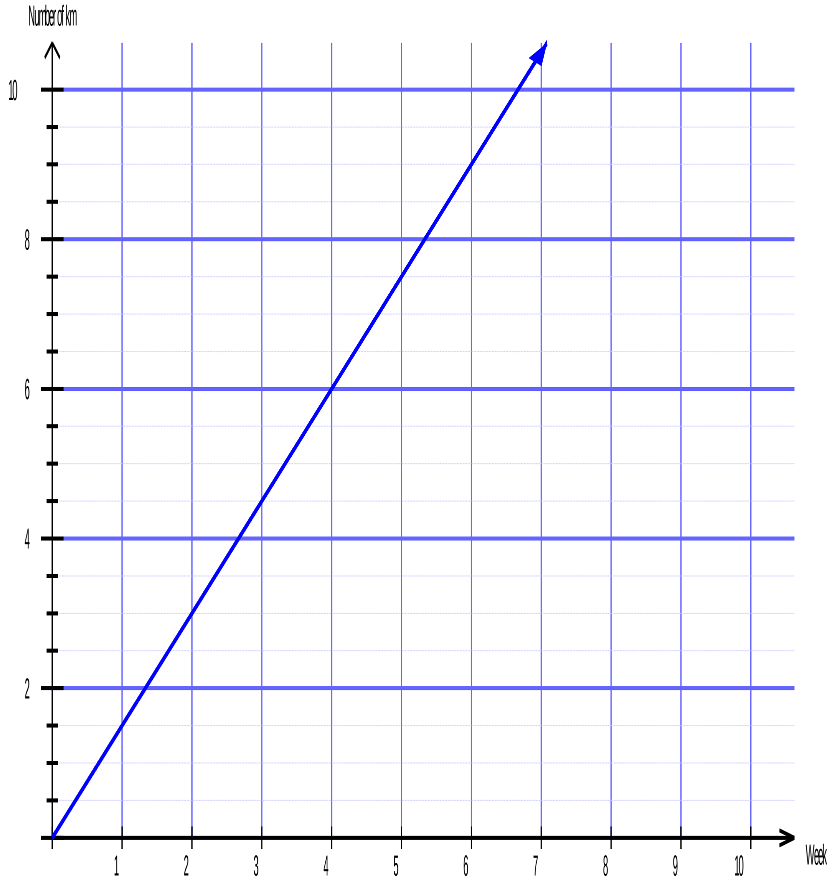
a) Determine the rate at which the bacteria is growing each hour.

b) After how many hours will the number of bacteria first exceed 300 000?

c) What is the rule which can be used to calculate the number of bacteria, *B*, after *t* hours.

Question Six: [2, 1, 1, 1: 5 marks]

Isabel is attempting to increase her fitness and has decided to follow a fitness program called *Sofa to Six*. The aim is to slowly build up her fitness to get her from doing nothing (sitting on the sofa) to running six kilometers. She records the distance she runs in total each week in the app on her phone and a graph is generated to show her progress.



a) Write the recursive rule to show how many km Isabel has run each week.

b) How many kilometers does Isabel run in the first week?

c) How long does it take for her to achieve the goal of running 6km?

d) How many kilometers has Isabel run in total after 6 weeks?

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Topic: Arithmetic and Geometric Sequences and Linear Recurrence
SOLUTIONS

Time: 45 mins   Marks: /45 marks

No calculator allowed

Question One: [2, 2, 2, 2, 2: 10 marks]

For the following sequences determine which are arithmetic, geometric or neither. Provide a reason to support your answer.

a) 1, 1.5, 2.25, 3.375...

Gometric ✓

$\times 1.5$ ✓

b) 5, -5, 5, -5, 5, -5 ...

Geometric ✓

$\times -1$ ✓

c) $\frac{7}{8}, \frac{29}{24}, \frac{37}{24}, \frac{15}{8}, \frac{53}{24} \dots$

$\frac{21}{24}$ $\frac{45}{24}$ Arithmetic ✓

$+\frac{8}{24}$ ✓

d) 300, -60, 12, -2.4, 0.48, -0.096 ...

Geometric ✓

$\times -\frac{1}{5}$ ✓

e) 2, 1, 2, 1, 2, 1 ...

Neither ✓

$-1 \; then + 1$ ✓

Question Two: [3, 3, 3: 9 marks]

a) A geometric sequence has $T_3 = 4$ and $T_6 = 32$.

i) Determine the recursive rule.

$4 \times r^3 = 32$

$r^3 = 8$

$r = 2$ ✓

$T_2 = 2 \; T_1 = 1$

$T_{n+1} = 2 \times T_n \qquad T_1 = 1$ ✓

ii) Calculate the 5th term.

$T_5 = 32 \div 2 = 16$ ✓

b) An arithmetic sequence has $T_3 = -5$ and $T_6 = 4$.

i) Determine the recursive rule.

$-5 + 3d = 4$

$3d = 9$

$d = 3$ ✓

$T_2 = -8 \; T_1 = -11$

$T_{n+1} = T_n + 3 \quad T_1 = -11$ ✓

ii) Calculate the 5th term.

$T_5 = 1$ ✓

c) For the following sequence determine the recursive rule and term and T_7.

T_1	T_2	T_3	T_4	T_5
4	-8	16	-32	64

$r = -2 \qquad T_{n+1} = -2 \times T_n \quad T_1 = 4$ ✓✓

$T_6 = -128 \qquad T_7 = 256$ ✓

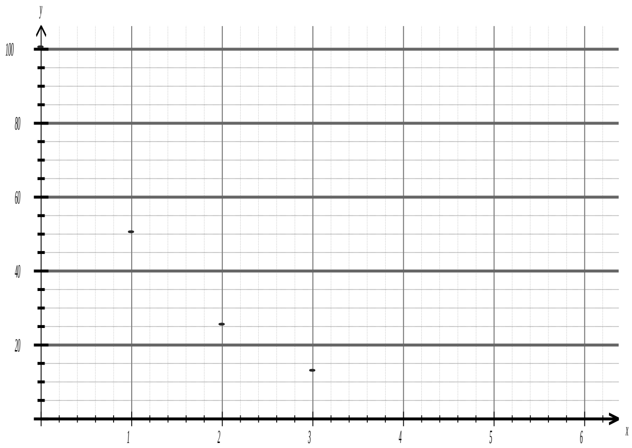
Question Three: [3, 2, 2: 7 marks]

a) An arithmetic sequence has $T_3 = 9$ and a common difference of 2. Determine term 12.

$$T_1 = 5 \quad T_n = 5 + 2(n-1)$$

$$T_{12} = 27$$

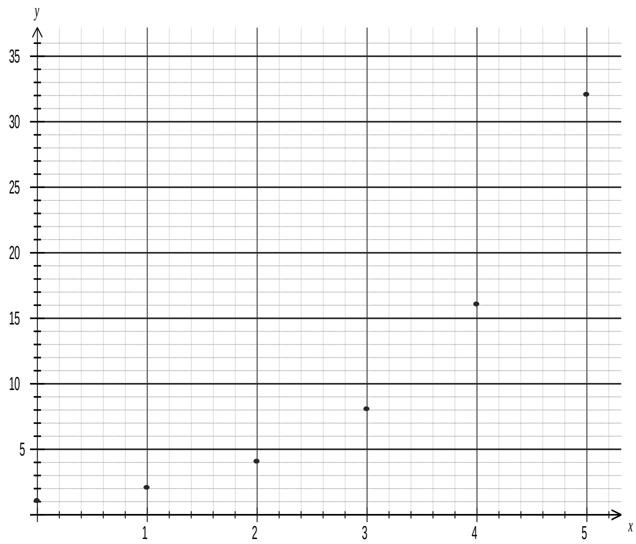
b) The following graph depicts a geometric sequence.



Determine the rule to find the n^th term.

$T_0 = 100\ T_1 = 50\ T_2 = 25 \ldots$ $\qquad T_n = 100 \times \frac{1}{2}^n \quad or \quad T_n = 50 \times \left(\frac{1}{2}\right)^{n-1}$

c) The following graph depicts a geometric sequence.



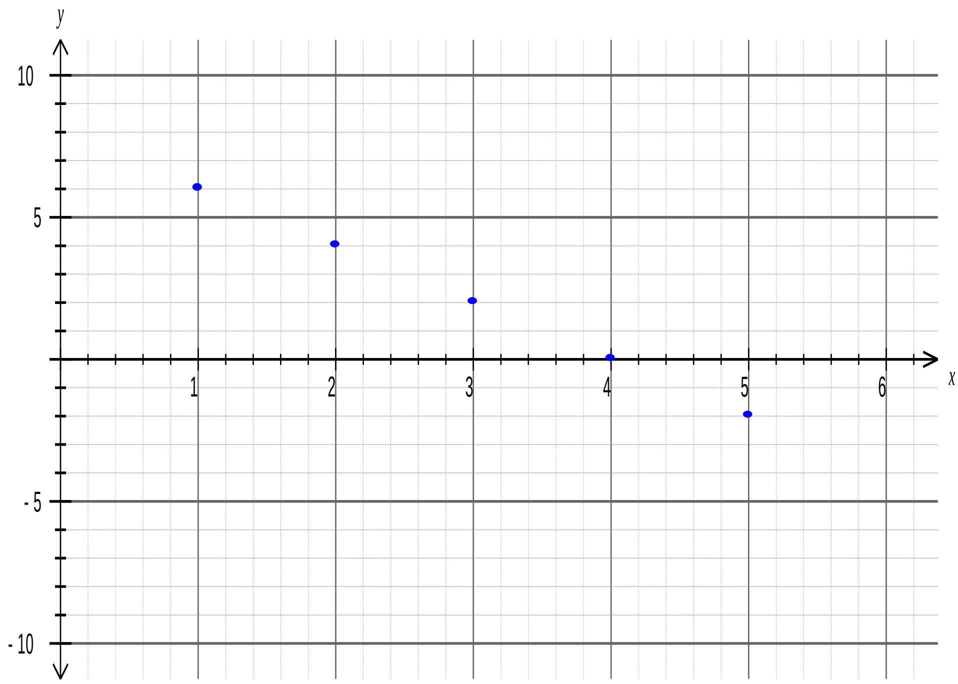
Determine the 8^th term.

$T_0 = 1\ T_1 = 2\ T_2 = 4 \ldots \qquad T_5 = 32 \qquad T_8 = 256$

Question Four: [4, 4: 8 marks]

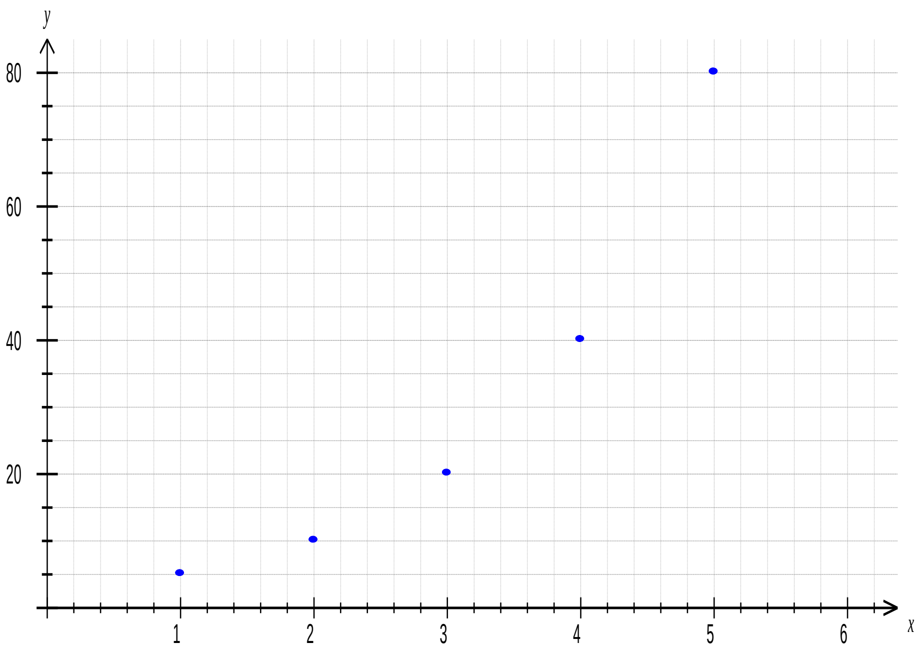
a) Generate the first 5 terms according to the following recursive rule and graph the terms on the axis below.

$T_{n+1} = T_n - 2, T_1 + T_2 = 10$ $T_1 = 6$ $T_2 = 4$ $T_3 = 2$ $T_2 = 0$ $T_1 = -2$



b) Graph the first 5 terms generated by the following recursive rule on the axis below.

$T_n = 2T_{n-1}$, $T_1 + T_2 = 15$ $T_1 = 5$ $T_2 = 10$ $T_3 = 20$ $T_4 = 40$ $T_5 = 80$



Question Five: [2, 2, 2: 6 marks]

Doctors are monitoring the growth of a bacteria in order to help calculate the dosage of medication needed to combat it. The following table shows the growth of the bacteria over several hours.

Number of bacteria	Number of hours
100	0
120	1
144	2
172.8	3
207.36	4

a) Determine the rate at which the bacteria is growing each hour.

$120 \div 100 = 1.2 \quad r = 1.2$ ✓ ✓

b) After how many hours will the number of bacteria first exceed 300?

$\begin{array}{r} 207.36 \\ \times\ 1.2 \\ \hline 248.832 \end{array} \quad \begin{array}{r} 248.832 \\ \times\ 1.2 \\ \hline 298.5984 \end{array} \quad \begin{array}{r} 298.5984 \\ \times\ 1.2 \\ \hline 358.31808 \end{array} \quad \therefore After\ 7\ hours$ ✓

✓

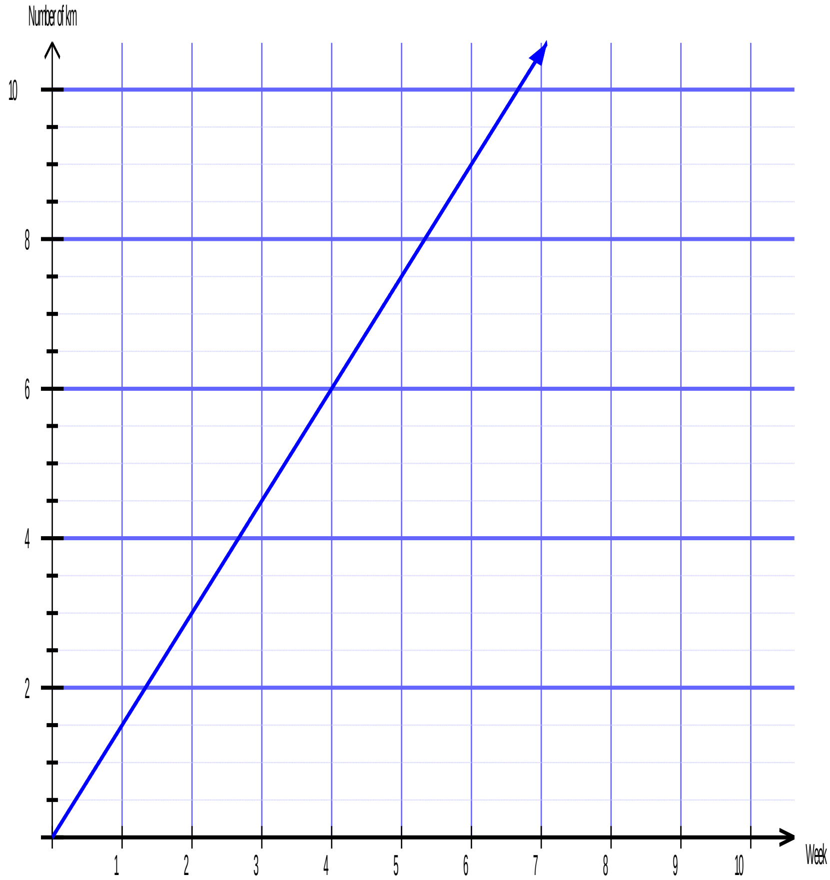
c) What is the rule which can be used to calculate the number of bacteria B after t hours.

$B = 100 \times 1.2^t \qquad or \qquad B = 120 \times 1.2^{t-1}$

✓ ✓

Question Six: [2, 1, 1, 1: 5 marks]

Isabel is attempting to increase her fitness and has decided to follow a fitness program called *Sofa to Six*. The aim is to slowly build up her fitness to get her from doing nothing (sitting on the sofa) to running six kilometers. She records the total distance she runs runs each week in the app on her phone and a graph is generated to show her progress.



a) Write the recursive rule to show how many km Isabel has run each week.

$$T_{n+1} = T_n + 1.5, \;\; T_1 = 1.5$$

b) How many kilometers does Isabel run in the first week?

$1.5\ km$

c) How long does it take for her to achieve the goal of running 6km?

$4\ weeks$

d) How many kilometers has Isabel run in total after 6 weeks?

$$1.5 + 3 + 4.5 + 6 + 7.5 + 9 = 31.5\ km$$